Representation Theory of Finite Groups

Semestral Examination

November 13, 2024

Instructions: All questions carry ten marks. Vector spaces and representations are assumed to be finite dimensional over the field of complex numbers.

1. Let Let σ be an even permutation in the permutation group S_d. Let $\sigma = \tau_1\tau_2\cdots\tau_k$ be its disjoint cycle decomposition. Prove that the conjugacy class of σ in S_d splits as a union of two conjugacy classes in the alternating subgroup A_d if and only if all τ_i are of odd length, and no two of them are of same length.
2. Prove that the alternating group A_4 has exactly four mutually non-isomorphic irreducible representations. Further show that three of those are one-dimensional and compute the character table of A_4.
3. Let H be a subgroup of a finite group G. Prove that the regular representation of G is induced from the regular representation H.
4. Let V be an irreducible representation of a finite group G. Determine all G-equivariant linear operators of V.
5. Let G be a finite group and χ be the character of an irreducible representation V of dimension d of G. Let $g \in G$. Prove that $\chi(g) = d$ if and only if g acts as identity on V.